Quiz 2, MATH 240, Fall 2023

Write your name clearly.

Name:

Section Number: UID:

(1) Consider the following system of equations:

$$x_1 + 3x_2 - x_3 = 4$$
$$-3x_1 - 9x_2 + x_3 = -2$$
$$2x_1 + 6x_2 - 4x_3 = 18$$

(a) [3] Express the system as the matrix equation $A\mathbf{x} = \mathbf{b}$.

(b) [12] Find the solution set to the system.

(c) [5] Write $\begin{pmatrix} 4 \\ -2 \\ 18 \end{pmatrix}$ as a linear combination of $\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -9 \\ 6 \end{pmatrix}$, and $\begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix}$. You should have numbers for your weights, no variables.

(a) We can write the system as $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 1 & 3 & -1 \\ -3 & -9 & 1 \\ 2 & 6 & -4 \end{pmatrix} \quad \text{and} \quad \vec{b} = \begin{pmatrix} 4 \\ -2 \\ 18 \end{pmatrix}$$

(b)

$$\begin{bmatrix} 1 & 3 & -1 & 4 \\ -3 & -9 & 1 & -2 \\ 2 & 6 & -4 & 18 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So the solutions are given by

$$x_1 = -1 - 3x_2$$
$$x_2 \text{ free}$$
$$x_3 = -5 \,.$$

(c) Let $x_2 = 0$. Then $x_1 = -1$. So

$$\begin{pmatrix} 4 \\ -2 \\ 18 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + 0 \begin{pmatrix} 3 \\ -9 \\ 6 \end{pmatrix} - 5 \begin{pmatrix} -1 \\ 1 \\ -4 \end{pmatrix}$$